

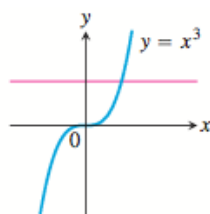
# Section 1.5

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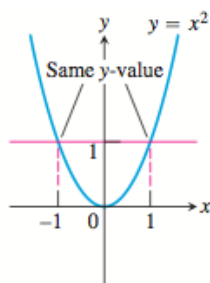
**DEFINITION One-to-One Function**

A function  $f(x)$  is **one-to-one** on a domain  $D$  if  $f(a) \neq f(b)$  whenever  $a \neq b$ .

The graph of a one-to-one function  $y = f(x)$  can intersect any horizontal line at most once (the *horizontal line test*). If it intersects such a line more than once it assumes the same  $y$ -value more than once, and is therefore not one-to-one (Figure 1.33).



One-to-one: Graph meets each horizontal line once.



Not one-to-one: Graph meets some horizontal lines more than once.

**Figure 1.33** Using the horizontal line test, we see that  $y = x^3$  is one-to-one and  $y = x^2$  is not.

1 Answer

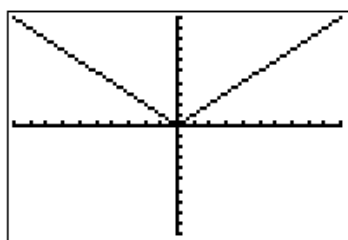
Yes

No

**EXAMPLE 1 Using the Horizontal Line Test**

Determine whether the functions are one-to-one.

(a)  $f(x) = |x|$



Graph

no

2 Answer?

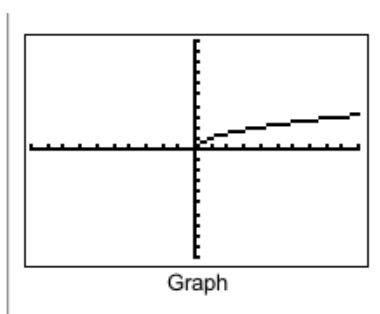
Yes

No

**EXAMPLE 1 Using the Horizontal Line Test**

Determine whether the functions are one-to-one.

(b)  $g(x) = \sqrt{x}$



yes

## Inverses

**Table 1.13** Rental Charge  
versus Time

Time $x$ (hours)	Charge $y$ (dollars)
1	5.00
2	7.50
3	10.00
4	12.50
5	15.00
6	17.50

**Table 1.14** Time versus  
Rental Charge

Charge $x$ (dollars)	Time $y$ (hours)
5.00	1
7.50	2
10.00	3
12.50	4
15.00	5
17.50	6

As Tables 1.13 and 1.14 suggest, composing a function with its inverse in either order sends each output back to the input from which it came. In other words, the result of composing a function and its inverse in either order is the **identity function**, the function that assigns each number to itself. This gives a way to test whether two functions  $f$  and  $g$  are inverses of one another. Compute  $f \circ g$  and  $g \circ f$ . If  $(f \circ g)(x) = (g \circ f)(x) = x$ , then  $f$  and  $g$  are inverses of one another; otherwise they are not. The functions  $f(x) = x^3$  and  $g(x) = x^{1/3}$  are inverses of one another because  $(x^3)^{1/3} = x$  and  $(x^{1/3})^3 = x$  for every number  $x$ .

### EXPLORATION 1 Testing for Inverses Graphically

For each of the function pairs below,

(a) Graph  $f$  and  $g$  together in a square window.

(b) Graph  $f \circ g$ . (c) Graph  $g \circ f$ .

What can you conclude from the graphs?

- $f(x) = x^3$ ,  $g(x) = x^{1/3}$  **yes**
- $f(x) = x$ ,  $g(x) = 1/x$  **no**
- $f(x) = 3x$ ,  $g(x) = x/3$  **yes**
- $f(x) = e^x$ ,  $g(x) = \ln x$  **yes**

**Writing  $f^{-1}$  as a Function of  $x$** 

1. Solve the equation  $y = f(x)$  for  $x$  in terms of  $y$ .
2. Interchange  $x$  and  $y$ . The resulting formula will be  $y = f^{-1}(x)$ .

**EXAMPLE 2 Finding the Inverse Function**

Show that the function  $y = f(x) = -2x + 4$  is one-to-one and find its inverse function.

$$y = -2x + 4$$

$$x = -2y + 4$$

$$x - 4 = -2y$$

$$y = -\frac{1}{2}x + 2$$

**DEFINITION Base  $a$  Logarithm Function**

The **base  $a$  logarithm function**  $y = \log_a x$  is the inverse of the base  $a$  exponential function  $y = a^x$  ( $a > 0, a \neq 1$ ).

**Inverse Properties for  $a^x$  and  $\log_a x$** 

1. Base  $a$ :  $a^{\log_a x} = x$ ,  $\log_a a^x = x$ ,  $a > 1, x > 0$
2. Base  $e$ :  $e^{\ln x} = x$ ,  $\ln e^x = x$ ,  $x > 0$

Solve for  $x$ :

$$\begin{aligned} \text{(a) } \ln x &= 3t + 5 && \text{power} \\ &&& \downarrow \\ \log_e x &= 3t + 5 && \text{Base} \\ e^{3t+5} &= x \end{aligned}$$

3 Answer (nearest hundredth)?

$$(b) e^{2x} = 10$$

$$\ln 10 = 2x$$

$$\frac{1}{2} \ln 10 = x$$



### Properties of Logarithms

For any real numbers  $x > 0$  and  $y > 0$ ,

1. *Product Rule:*  $\log_a xy = \log_a x + \log_a y$
2. *Quotient Rule:*  $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. *Power Rule:*  $\log_a x^y = y \log_a x$

### EXPLORATION 2 Supporting the Product Rule

Let  $y_1 = \ln(ax)$ ,  $y_2 = \ln x$ , and  $y_3 = y_1 - y_2$ .

1. Graph  $y_1$  and  $y_2$  for  $a = 2, 3, 4$ , and  $5$ . How do the graphs of  $y_1$  and  $y_2$  appear to be related?
2. Support your finding by graphing  $y_3$ .
3. Confirm your finding algebraically.

$$\begin{aligned}
 a &= 2 \\
 y &= \ln 2x - \ln x \\
 &= \ln \frac{2x}{x} \\
 &= \ln 2 \\
 y &\approx .7
 \end{aligned}$$

**Change of Base Formula**

$$\log_a x = \frac{\ln x}{\ln a} \quad \frac{\log_{10} x}{\log_{10} a}$$

**EXAMPLE 5 Graphing a Base  $a$  Logarithm Function**

Graph  $f(x) = \log_2 x$ .

$$= \frac{\ln x}{\ln 2}$$

4 Answer (nearest tenth in years)?

**EXAMPLE 6 Finding Time**

Sarah invests \$1000 in an account that earns 5.25% interest compounded annually. How long will it take the account to reach \$2500?

$$\text{amount} = \text{initial} (1 + r)^{t(1)}$$

↓ how many  
times a year

$$2500 = 1000 (1.0525)^t$$

$$1.0525^t = 2.5$$

$$\log_{1.0525} 2.5 = t$$

$$\frac{\ln 2.5}{\ln 1.0525} = t$$

5 Answer (nearest hundredth in trillion cubic feet)?

**EXAMPLE 7 Estimating Natural Gas Production**

Table 1.15 shows the annual number of cubic feet in trillions of natural gas produced by Saudi Arabia for several years.

Find the natural logarithm regression equation for the data in Table 1.15 and use it to estimate the number of cubic feet of natural gas produced by Saudi Arabia in 2002. Compare with the actual amount of 2.00 trillion cubic feet in 2002.

**Table 1.15 Saudi Arabia's  
Natural Gas Production**

Year	Cubic Feet (trillions)
1997	1.60
1998	1.65
1999	1.63
2000	1.76
2001	1.90

*Source: Statistical Abstract of the  
United States, 2004–2005.*

# Section 1.6

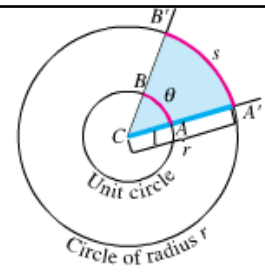
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## Radian Measure

The **radian measure** of the angle  $ACB$  at the center of the unit circle (Figure 1.40) equals the length of the arc that  $ACB$  cuts from the unit circle.

### EXAMPLE 1 Finding Arc Length

Find the length of an arc subtended on a circle of radius 3 by a central angle of measure  $2\pi/3$ .



**Figure 1.40** The radian measure of angle  $ACB$  is the length  $\theta$  of arc  $AB$  on the unit circle centered at  $C$ . The value of  $\theta$  can be found from any other circle, however, as the ratio  $s/r$ .

1 Answer (nearest hundredth)?

**sine:**  $\sin \theta = \frac{y}{r}$

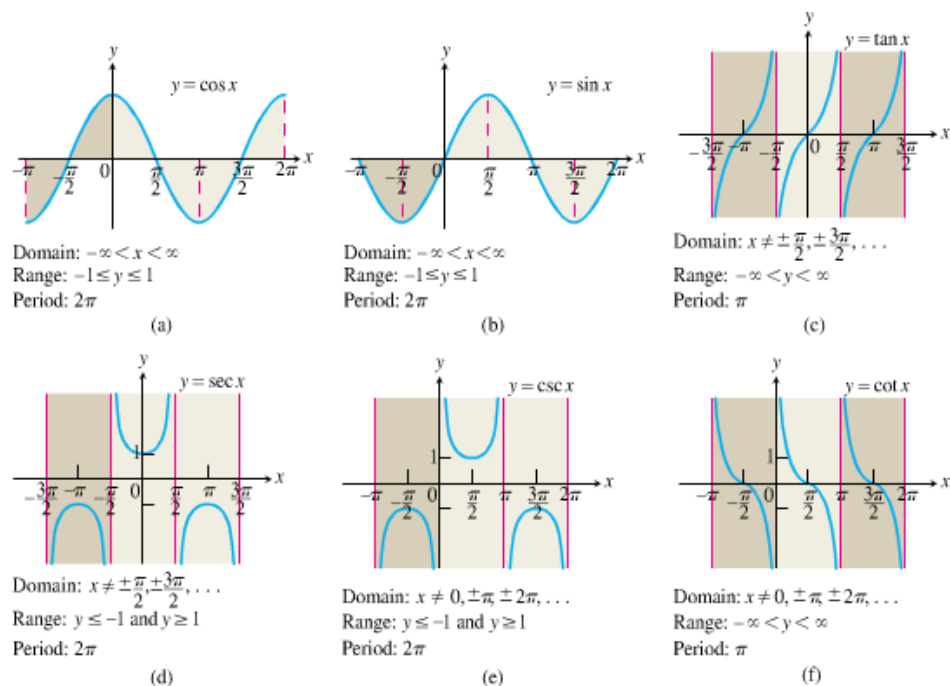
**cosecant:**  $\csc \theta = \frac{r}{y}$

**cosine:**  $\cos \theta = \frac{x}{r}$

**secant:**  $\sec \theta = \frac{r}{x}$

**tangent:**  $\tan \theta = \frac{y}{x}$

**cotangent:**  $\cot \theta = \frac{x}{y}$



**Figure 1.42** Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure.

Graph these functions on your grapher and compare with the sketches. Use Zoom 7 for your window.



**EXAMPLE 3 Finding Trigonometric Values**

Find all the trigonometric values of  $\theta$  if  $\sin \theta = -3/5$  and  $\tan \theta < 0$ .

2 Cosine?

3 Tangent?

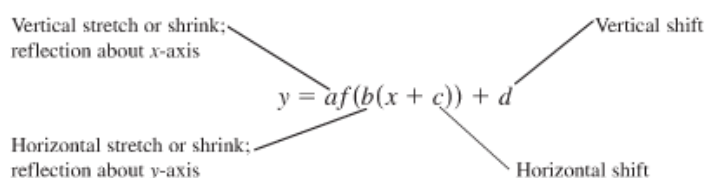
#### 4 Cosecant?

5 Secant?

6 Cotangent?

## Transformations of Trigonometric Graphs

The rules for shifting, stretching, shrinking, and reflecting the graph of a function apply to the trigonometric functions. The following diagram will remind you of the controlling parameters.



The general sine function or **sinusoid** can be written in the form

$$f(x) = A \sin \left[ \frac{2\pi}{B}(x - C) \right] + D,$$

where  $|A|$  is the *amplitude*,  $|B|$  is the *period*,  $C$  is the *horizontal shift*, and  $D$  is the *vertical shift*.

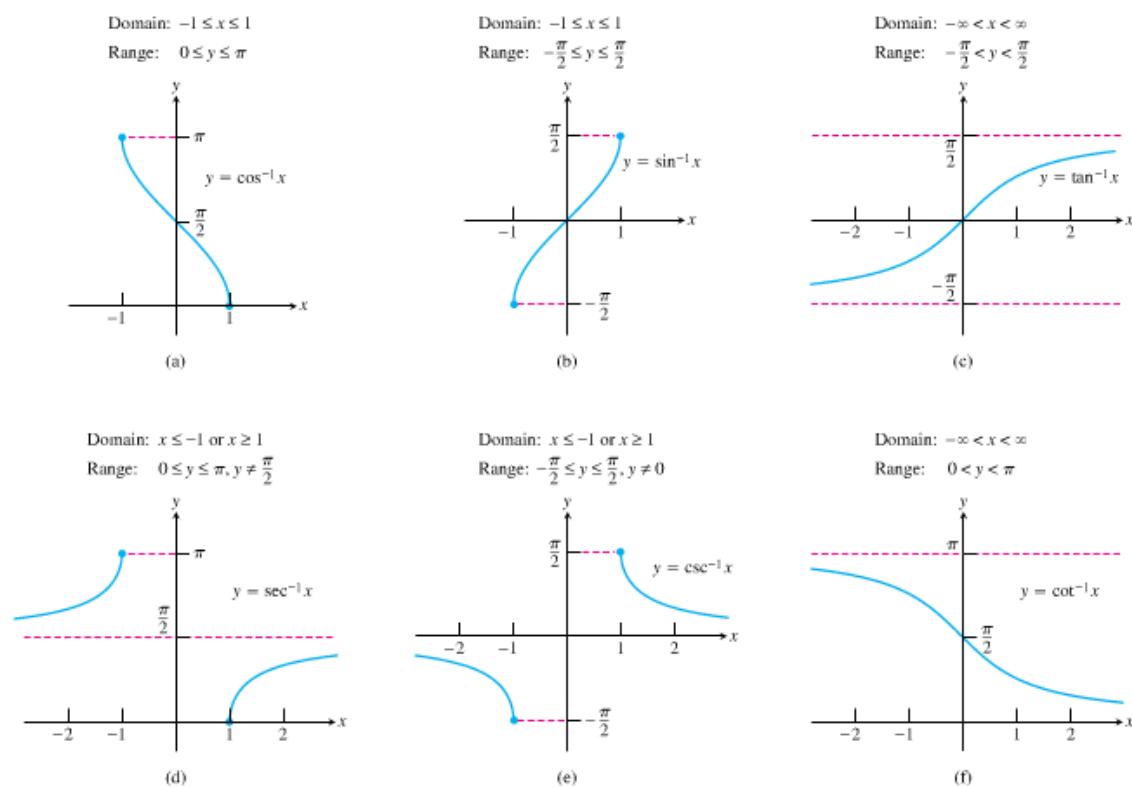
### EXAMPLE 4 Graphing a Trigonometric Function

Determine the (a) period, (b) domain, (c) range, and (d) draw the graph of the function  $y = 3 \cos(2x - \pi) + 1$ .

Use GSP to understand this function.

**DEFINITIONS Inverse Trigonometric Functions**

Function	Domain	Range
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$	$ x  \geq 1$	$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$
$y = \csc^{-1} x$	$ x  \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$

**Figure 1.48** Graphs of (a)  $y = \cos^{-1} x$ , (b)  $y = \sin^{-1} x$ , (c)  $y = \tan^{-1} x$ , (d)  $y = \sec^{-1} x$ , (e)  $y = \csc^{-1} x$ , and (f)  $y = \cot^{-1} x$ .

7 Answer #1(nearest thousandth)?

**EXAMPLE 8** Using the Inverse Trigonometric Functions

Solve for  $x$ .

(a)  $\sin x = 0.7$  in  $0 \leq x < 2\pi$



8 Answer #2(nearest thousandth)?

**EXAMPLE 8** Using the Inverse Trigonometric Functions

Solve for  $x$ .

(a)  $\sin x = 0.7$  in  $0 \leq x < 2\pi$

$$(b) \tan x = -2 \text{ in } -\infty < x < \infty$$